

TABLE I
ACRONYMS

Acro

[Redacted content]

In [48]–[50], the AM-FM decomposition of a signal $f(t)$ is given by $f(t) = A(t) \cos(\phi(t))$, where $A(t)$ is the amplitude and $\phi(t)$ is the phase. The AM-FM decomposition is used in [51], [52], and [40] to decompose a signal into its amplitude and phase components. The AM-FM decomposition is also used in [53] to decompose a signal into its amplitude and phase components. The AM-FM decomposition is also used in [54] to decompose a signal into its amplitude and phase components.

$$1(\omega_1, \omega_2) = \frac{-1}{2\pi \left(\frac{\omega_1}{2} + \frac{\omega_2}{2}\right)^{3/2}}$$

$$2(\omega_1, \omega_2) = \frac{-2}{2\pi \left(\frac{\omega_1}{2} + \frac{\omega_2}{2}\right)^{3/2}} \quad (1)$$

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$$C(w) = c|w|^a \quad (-, |w|), a \geq 1 \quad (2)$$

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$$PA = \sum \frac{[|e| - |e| - T]}{\sqrt{e^2 + \dots + \varepsilon}} \quad (3)$$

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Fig. 1. Example of PAS decomposition of a signal $f(t)$. The signal $f(t)$ is decomposed into its amplitude $A(t)$ and phase $\phi(t)$ components. The PAS decomposition is used in [51], [52], and [40] to decompose a signal into its amplitude and phase components. The PAS decomposition is also used in [53] to decompose a signal into its amplitude and phase components. The PAS decomposition is also used in [54] to decompose a signal into its amplitude and phase components.

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B. Fractional-Order Derivative

The fractional-order derivative of a function $f(t)$ is given by $D^\alpha f(t)$, where α is the order of the derivative. The fractional-order derivative is used in [47] to decompose a signal into its amplitude and phase components. The fractional-order derivative is also used in [53] to decompose a signal into its amplitude and phase components. The fractional-order derivative is also used in [54] to decompose a signal into its amplitude and phase components.

$$D^\alpha f(t) = \frac{d^\alpha f(t)}{dt^\alpha} \quad (4)$$

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$$D^\alpha f(t) \stackrel{FT}{\leftrightarrow} (\hat{D}^\alpha f)(w) = (w)^\alpha \hat{f}(w)$$

$$= |w|^\alpha \left[\theta^\alpha(w) \right] \hat{f}(w)$$

$$= |w|^\alpha \left[\frac{\alpha\pi}{2}, n(w) \right] \hat{f}(w) \quad (5)$$

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Fig. 2. Total variation (TV) denoising results for the original image f and its denoised version f_{TV} .

The TV denoising results for the original image f and its denoised version f_{TV} are shown in Fig. 2. The original image f is a grayscale image of a textured surface. The denoised version f_{TV} shows significant improvement in image quality, with reduced noise and enhanced edge detection. The TV denoising process is based on the total variation functional, which is defined as the integral of the magnitude of the gradient of the image. The TV denoising process is a non-linear process, and it is known to be effective in removing noise while preserving edges. The TV denoising process is based on the total variation functional, which is defined as the integral of the magnitude of the gradient of the image. The TV denoising process is a non-linear process, and it is known to be effective in removing noise while preserving edges.

$$D^\alpha f(x) \triangleq \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{k=0}^{[d-c]} (-1)^k \binom{\alpha}{k} f(x - kh) \quad (6)$$

where $\alpha \in [0, 1]$, $f \in C^{\alpha, \alpha}$ on $[c, d]$, and $f(x) = 0$ for $x < c$ and $x > d$. The binomial coefficient $\binom{\alpha}{k}$ is defined as

$$\binom{\alpha}{k} = \frac{\Gamma(\alpha + 1)}{\Gamma(k + 1)\Gamma(\alpha - k + 1)} \quad (7)$$

where $\Gamma(\cdot) = (\cdot - 1)!$ for $\cdot \in \mathbb{N}$.

C. Fractional Anisotropic Diffusion (FAD) and Fractional Total Variation (FTV) Denoising

The FAD denoising process is based on the fractional anisotropic diffusion equation, which is a partial differential equation that models the diffusion of a scalar field. The FAD denoising process is based on the fractional anisotropic diffusion equation, which is a partial differential equation that models the diffusion of a scalar field.

$$\frac{\partial f}{\partial t} = \text{div} [c(|\nabla f|) \cdot \nabla f], \quad (8)$$

where div is the divergence operator, $c(\cdot)$ is a diffusion coefficient function, and ∇f is the gradient of f .

TABLE II
COMPARISON OF THE PSNR, MSSIM AND FSIM VALUES AMONG DIFFERENT FILTERS



(a) (b) (c) (d)

Fig. 9. Median values for different segmentation methods on ten breast ultrasound images. (a) F, (b) SRAD, (c) OBNLM, (d) SBF, (e) ADLG, (f) NLLRE, (g) PFDTV.

TABLE IV
THE MEDIAN DSC, JS, HD AND HM VALUES FOR DIFFERENT SEGMENTATION RESULTS ON TEN BREAST ULTRASOUND IMAGES

In [2], ACVA, P, T, [78], [79] PFDTV

[35], [76], [80] M, 3D, [7], [81] H w

T, w, [82] f, w

T, [39] f, w, Fr, w, [38], [39]

In TV, FAD, FTV, S, TV, w, n

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T, w, n, w

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